Program II. **RELATIONS AND FUNCTIONS**

SOL Topic:

A. 16

The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of f(X) will be related to the ordinate on the graph.

Activity 1: **Solve the equation 3x - 15 = 12 on the graphing calculator using four methods of inquiry:**

- Let $Y_1 = 3x - 15$, **Trace** on an integer window (zoom 8) until y = 12, read the x-value
- Examine the **table values** where y= 12, READ the corresponding x element
- Using the **multi-graph method**, let $Y_1 = 3x - 15$, $Y_2 = 12$, using the multi-graph method find the point of intersection, read the x-value for the solution to the equation
- Subtracting 12 from both side of the equation, we obtain 3x - 27 = 0. Let $Y_1 = 3x - 27$. Observe where the equation crosses the x- axis. The x-value is the solution to the equation. This is the **x-intercept method**.

SOL Topic:

A. 14

The student will solve quadratic equations in one variable both algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

Activity 2: **Solving quadratic equations: Using the same four methods of solution on the Graphing calculator, Solve $x^2 - 2x - 13 = 2$.**

- Trace Method
- Multigraph Method
- Table Values Method
- X-intercept Method

SOL Topic:
AII.8

The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, table, and symbolic form. A transformation approach to graphing will be employed through the use of graphing calculators.

Activity 3: **Transformations of quadratic equations:**
Question--How does the graph change in shape, domain and range?

- Investigate the role of "K" in $Y = X^2 + K$
- Investigate the role of "H" in $Y = (X - H)^2$
- Investigate the role of "A" in $Y = A*X^2$
- Predict and describe the graph of $Y = 2(X + 4)^2 - 8$

Activity 4: **Transformations of Absolute Value Equations**

type these into $Y_1 =$ and $Y_2 =$ respectively

- Show: $Y_1 = abs\ (X)$
- $Y_2 = A*Y_1\ (X - H) + K$

Use this to generalize transformations. Store values for A, H, and K from the home screen. This is also know as Composition of Functions.
Examine:

- Exponential $Y = A*B^{(X - H)} + K$
- Logarithmic $Y = A*\log\ (X - H) + K$

- Step (Greatest Integer Function) Y = A*INT (X - H) + K

SOL Topic:

AII.15

The student will recognize the general shape of polynomial functions, locate the zeros, sketch the graphs, and verify graphical solutions algebraically. The graphing calculator will be used as a tool to investigate the shape and behavior of polynomial functions.

Activity 5: **Investigate the general shape of polynomial functions.**

The domain of all nonrestricted polynomials is negative infinity to positive infinity.

- $Y_1 = x$
- $Y_2 = x^2$
- $Y_3 = x^3 - 4x$
- $Y_4 = x^4 + x^3 - 4x^2 - 4x$

Range for odd degree polynomial is negative infinity to positive infinity

Range for even degree polynomial is:

- from the minimum point to positive infinity or
- from negative infinity to the maximum point

Activity 6*:* Locating the zeros, sketch the graphs, and verify graphical solutions algebraically.

For : $y = x^3 - 3x^2 - 4x + 12$ use the Rational Root Theorem to find the zeros.

P = { -12, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6, 12}

Q = {-1, 1}

P/Q = { 12, 6, 4, 3, 2, 1, -1, -2, -3, -4, -6, -12} are the possible rational roots

Use the Graph to eliminate some roots and to estimate possible rational roots :

- $Y_1 = x^3 - 3x^2 - 4x + 12$, observe where f(x) = 0
- Use the CALC menu to find the zeros.
- Verify zeros algebraically on the home screen using $Y_1(x)$.
- Explore the TABLE values, show where y = 0.
- $Y_2 = (x - 2)(x - 3)(x + 2)$

Investigate non-complex, irrational roots using the graphing calculator.

- $Y_1 = x^3 - 3x^2 - 4x + 11$
- Let $Y_2 = 0$
- Show where does $Y_1 = Y_2$, using the INTERSECT feature.

SOL Topic:

AII.9

The student will find the domain, range, zeros and inverse of a function, the value of a function for a given element in its domain, and the composition of multiple functions. Functions will include those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions, including exponential and logarithmic.

Activity 7: **Introduce Parametric Equations to show restrictions on the Domain and to show the inverse of functions. Comparisons will be made to show the advantages of using parametric mode to show inverse functions.**

- **Graph $F(x) = x^2 - 4$ in Function Mode**
- **Graph $F(t) = t^2 - 4$ in Parametric Mode**

Investigate: In Parametric Mode, show Domain and Range Restrictions

- $X_{1t} = T$
- $Y_{1t} = T^2 - 4$

Where $T_{min} = 0$, $T_{max} = 4$, $T_{step} = .1$, the domain is restricted,
$X_{min} = -10$, $X_{max} = 10$, $X_{scl} = 1$, $Y_{min} = -10$, $Y_{max} = 10$, $Y_{scl} = 1$
denoted by $T : [0, 4]_{.1}$ $X : [-10, 10]_1$ $Y: [-10, 10]_1$

- Discuss domain and range
- Observe graphs and numerical representations
- Change T_{min} to - 3. Observe the changes in the graph.
- Discuss new graphs and tables.

Investigate inverse of function. For $T:[-10,10]_1$, $X: [-10, 10]_1$, $Y: [-10, 10]_1$

- Let $Y_3 = T^2 - 4$
- Let $Y_4 = T$

Show the parametric equivalent to the function "Y = X" line, the axis of symmetry.

- Let $Y_5 = T$
- Let $Y_6 = T$

Activity 8: **Investigate Composition of Functions:**

For $f(x) = x - 5$
$g(x) = x^2 + 3$
Investigate: $f \circ g(3) = f(g(3))$

- $Y_1 = x - 5$
- $Y_2 = x^2 + 3$

On home screen show that $Y_1(Y_2(3)) = 7$

On the graph screen: Show that these statements are equivalent.

- $Y_3 = Y_1(Y_2(x))$
- $Y_4 = ((x^2 + 3) - 5)$ or $Y_4 = x^2 - 2$
- Show the commutative properties of "f + g" and "g * f"
- Show the noncommutative properties of "f - g" and "g/f".

In the Zoom INTEGER Window:

- Emphasize the discontinuity in composition of "g / f".